Kuwait University, Faculty of Science
Dept. of Mathematics & Computer Science
Calculus I (Math 101) Second Mid-Term Test

Time: 75 min. May 6, 2004

Calculators, Mobile phones, Pagers and all other mobile communication equipments are NOT allowed.

Answer the following questions:

1. [4 points] Let $f(x) = 3(4+x)^{\frac{2}{3}}$. Use differentials to approximate $f(3.9)$.

2. [4 points] Find an equation of the tangent line to the graph of
$$\tan^2(x) - \cos(2x) - y^3 = 0$$
at the point whose x-coordinate is π.

3. [1+3 points]
   (a) State Rolle's theorem.
   (b) Let f, f' and f'' be continuous on $[a,b]$. Suppose that the graph of f intersects the x-axis at 3 points in (a,b). Show that $f''(x) = 0$ has a solution in (a,b).

4. [4 points] A snow ball is melting at a rate of $0.03 ft^3/hr$. At what rate is the surface area changing when the volume of the ball is 36π?

5. [9 points] Let $f(x) = \frac{3x^2-10x+3}{(x-1)^2}$.
   (a) Find the vertical and horizontal asymptotes for the graph of f, if any.
   (b) Given that $f'(x) = 4\,\frac{x+1}{(x-1)^3}$. Find the intervals on which f is increasing or decreasing and find the local extrema, if any.
   (c) Given that $f''(x) = -8\,\frac{x+2}{(x-1)^4}$. Find the intervals on which the graph of f is concave upward or concave downward and find the points of inflection, if any.
   (d) Is the graph of f symmetric with respect to the origin? Justify your answer.
   (e) Sketch the graph of f.

GOOD LUCK

1. $f(x) = 3(4+x)^{2/3}$, $f'(x) = 2(4+x)^{-1/3}$, $x = 4$, $\Delta x = -0.1$

$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

$f(3.9) \approx 12 - 0.1 = 11.9$

2. At $x = \pi$, $0 - 1 - y^3 = 0 \Rightarrow y = -1$.

$2\tan(x)\cdot\sec^2(x) + 2\sin(2x) - 3y^2 y' = 0$,

at $(\pi, -1)$, $y' = 0$. $\therefore$ T.L. $y = -1$

3. b) $f(c_1) = f(c_2) = f(c_3) = 0$; $a < c_1 < c_2 < c_3 < b$.

By the Rolle's theorem $\exists\ x_1$; $x_1 \in (c_1, c_2)$ and x_2; $x_2 \in (c_2, c_3)$ s.t. $f'(x_1) = f'(x_2) = 0$. By Rolle's theorem again; $f''(x) = 0$ for at least one number $x \in (x_1, x_2)$, $a < x < b$.

4. $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$, $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$, $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$.

$\frac{ds/dt}{dv/dt} = \frac{2}{r}$, when $V = 36\pi = \frac{4}{3}\pi r^3 \Rightarrow r = 3$.

$\therefore \frac{dS}{dt} = \frac{2}{3}\frac{dV}{dt} = (-)0.02$ ft²/hr.

5. a) V.A. $\lim_{x \to 1^+} f(x) = -\infty$. $x = 1$ is a V.A.

H.A. $\lim_{x \to \pm\infty} f(x) = 3$ $y = 3$ is a H.A.

b) Local maximum: $f(-1) = 4$

Local minimum: none.

max

Intervals	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Sign of f'	+	−	+
Conclusion	↗	↘	↗

c) P.I.: $(-2, \frac{35}{9})$.

Intervals	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of f''	+	−	−
Concavity	∪	∩	∩

d) $f(-x) = \frac{3x^2 + 10x + 3}{(-x-1)^2} \neq -f(x)$.

The graph of f is not symmetric with respect to the origin.